A Multimaterial Numerical Method for Eulerian Shock Physics

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Outline

- Introduction
- Motivation
- Governing equations
- Numerical method
- Examples
- Summary

Shock passage through heterogeneous media
Large number of mixed cells

Introduction

- Multimaterial Eulerian shock physics is an important class of problems
- Hydrocode implementations make various approximations to governing equations
- Focus: examine the assumptions made regarding computational cells with more than one material
- Seek resolution through governing equations
- Identify appropriate numerical methods

Motivational

- Most shock physics problems involve computational cells with more than one material (ALE or Eulerian)
- These cells lead to problems
  - Invalid density-energy states
  - Invalid velocity magnitude
  - Nonphysical results
- Current methods try to minimize problems
  - Mixture assumptions (P, T, ϕ)
  - Strain partitioning
  - Material discard

Motivational Continued

- Experimental and theoretical investigations are pushing the limits of the assumptions
- Advanced computational models are needed
  - Multiphase flow
  - Pressure/rate dependent strength
  - Heterogeneous media
  - Energetic material response
  - Porosity evolution
  - High energy density physics
  - Phase transition and kinetic processes
  - Fracture mechanics

Governing Equations

- **Hydrodynamic materials**
- **Unknowns: ϕ, γ, P, e, V (3+4m)**
- **Equations: 5 conservation, m EOS, m ϕ**
- **Definitions and constraints provide closure**
- **Critical point is cell pressure definition and energy partitioning into materials**

$$\frac{\partial}{\partial t}[\rho] + \nabla \cdot [\rho V] = 0$$

$$\frac{\partial}{\partial t}[\rho V] + \nabla \cdot [\rho V \otimes V + P] = 0$$

$$\frac{\partial}{\partial t}[\rho E] + \nabla \cdot [\rho E V + P V] = 0$$

$$\boxed{\frac{\partial}{\partial t}[\phi_i] + V \cdot \nabla \phi_i = 0}$$

or equivalent

$$\rho = \sum \phi_i \gamma_i$$

$$P = \sum \phi_i P_i$$

$$\sum \phi_i = 1$$

$$\rho E = \sum \phi_i \gamma_i (e_i + \frac{1}{2} V \cdot V)$$

Partitioning of Strain and Energy

1. Assume all materials have equal pressure and temperature
2. Same pressure different temperature
   - Iteratively adjust volume fraction and material energy subject to assumptions
3. Different pressure and temperature
   - Ad hoc rules for volume fraction change and cell pressure

- Consistent mass, momentum and energy

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Examples of Failure of Assumptions

- One pressure and temperature (solid in tension cannot exist in cell with gas)
- Pressure and temperature are not unique (phase transition) no viable or multiple EOS solutions
- Consider high pressure and low pressure in cell
  – Cell expansion applies to both materials equally
  – Expand both materials?
- High density and low density in same cell
  – Both experience same dilatation (strain)
  – High density material becomes nonphysical
  – Energy and density not consistent

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Failure is Associated with Missing Physics

- Heat Transfer
  - Materials at different temperature relax toward each other over time
- Pressure relaxation
  - Materials at different pressures relax toward each other to balance forces over time
- Configurational effects
  - Spatiotemporal scales, forces, chemistry, kinetics
  - Statistical materials and processes
- Fracture mechanics
  - Nature of fracture and mathematical/numerical implementation
- Momentum transport
  - Momentum transport in multimaterial cells

Addition to Governing Equations

- **Multiphase modeling suggest simplified extension for Eulerian hydrodynamics**
- **Additional volume fraction evolutionary equation**
- **Specific internal energy has a work term associated with volume fraction and configurational effects and heat transfer**
- **Highly coupled set of partial differential equations**

$$\frac{\partial}{\partial t}[\rho] + \nabla \cdot [\rho V] = 0$$

$$\frac{\partial}{\partial t}[\rho V] + \nabla \cdot [\rho V \otimes V + P] = 0$$

$$\frac{\partial}{\partial t}[\rho E] + \nabla \cdot [\rho E V + P V] = 0$$

$$\frac{\partial \phi_i}{\partial t} + V \cdot \nabla \phi_i = \frac{f(\phi)}{\mu_c}\left[P_i - \beta_i + P_j - \beta_j\right]$$

$$\phi_i \gamma_i \frac{\partial e_i}{\partial t} + \phi_i \gamma_i V \cdot \nabla e_i = \phi_i P_i \nabla \cdot V + P_{\mathrm{int}} \phi_i' + H(T_j - T_i)$$

$$\rho = \sum \phi_i \gamma_i$$

$$P = \sum \phi_i P_i$$

$$\sum \phi_i = 1$$

$$\rho E = \sum \phi_i \gamma_i (e_i + \frac{1}{2} V \cdot V)$$

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Numerical Solution Technique

- Apply method of fractional steps (Time splitting)
- Define three operators L_L,L_R and L_ϕ
- L_L performs Lagrangian hydrodynamics
  - Many appropriate methods and algorithms
- L_R performs Eulerian remap
- L_ϕ performs volume fraction evolution
- Order of operators
  - L_ϕ L_L L_ϕ L_R
  - L_ϕ L_L L_R
  - L_L L_ϕ L_R

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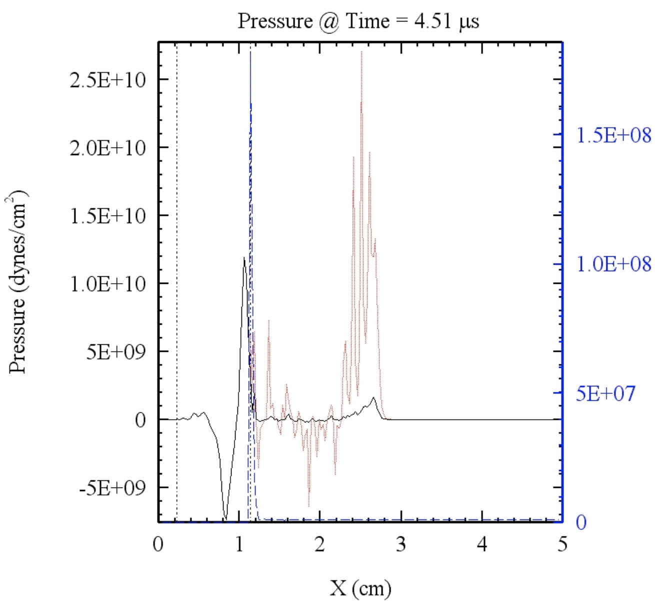
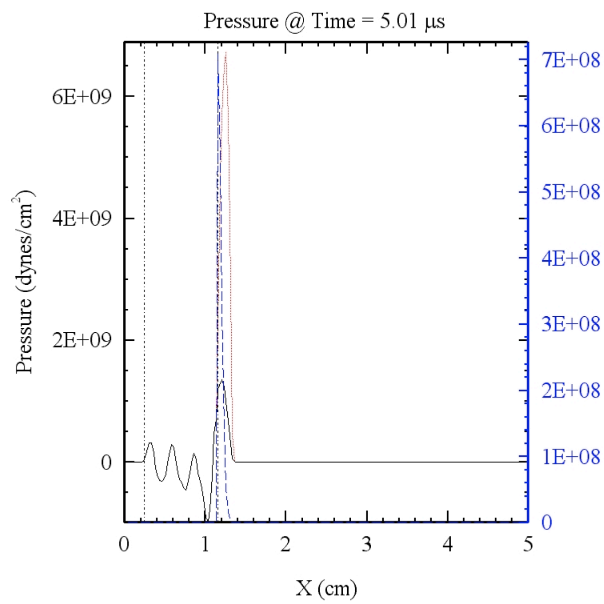
Extreme 1D Test Problem

Flyer Plate

No volume fraction
Unstable wave



With volume fraction
Stable wave



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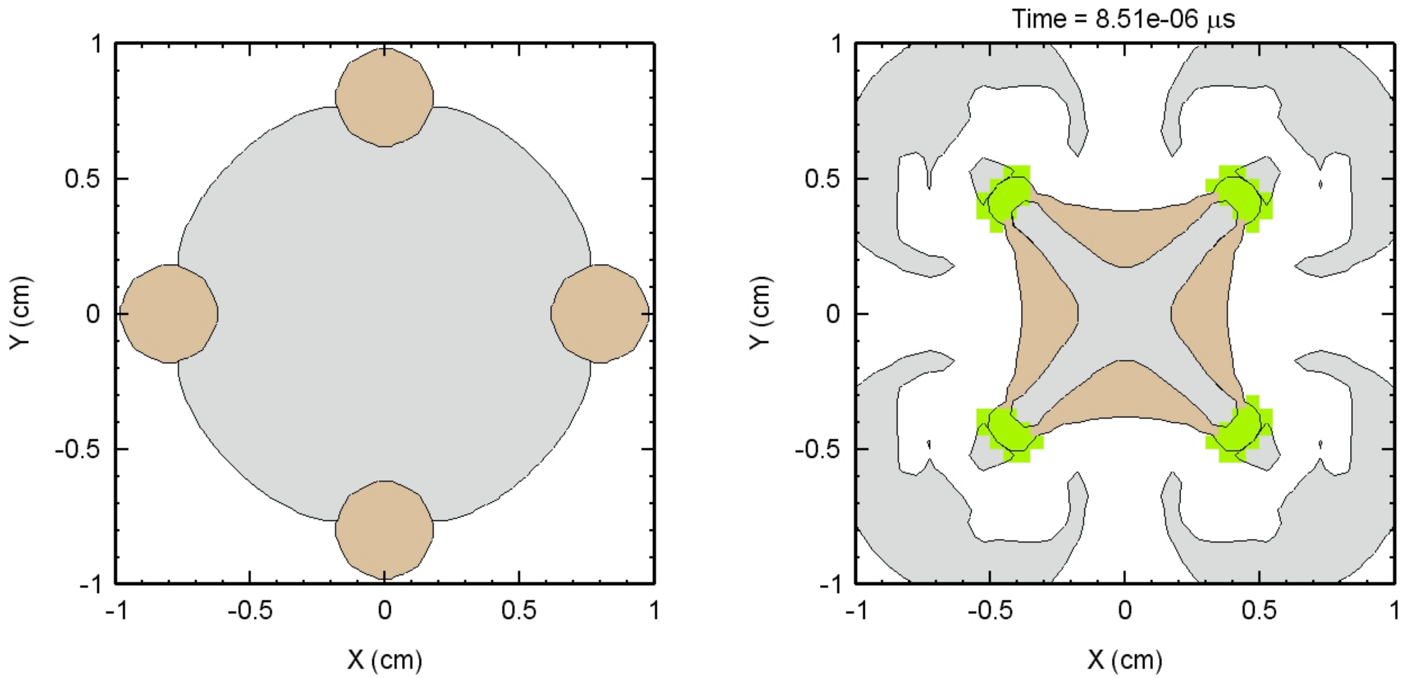
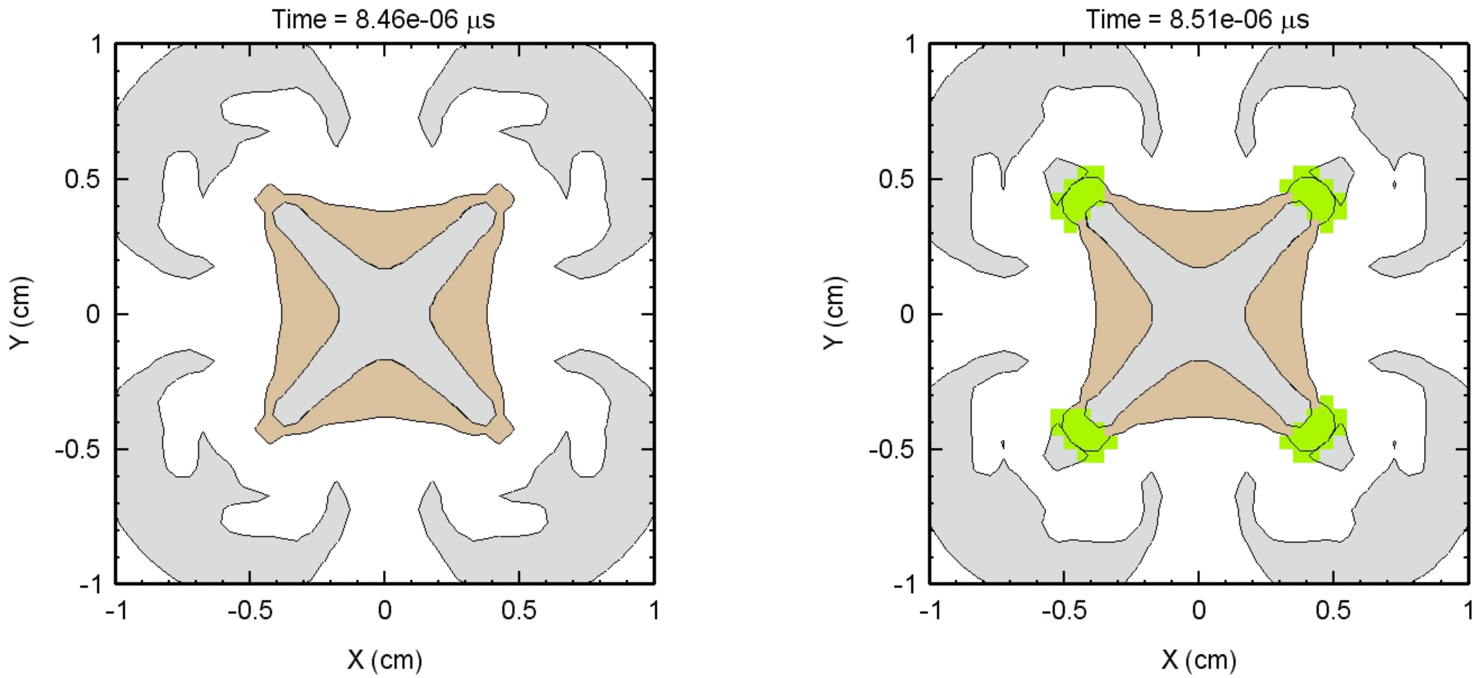
2D Test Problem



- Rods moving inward (1 km/s)
- 2000 Invalid density-energy states (no volume fraction)
- EOS representation in tension is questionable

2D Test Problem



- **Significantly reduced invalid density-energy states (12) with volume fraction evolution**
- **EOS representation in tension still problematic need to add missing fracture physics**
- **Solution character changes**

Summary and Conclusions

- Define problem class as mixed material cells
- Seek theoretical and numerical resolution
- Postulated volume fraction evolution equation
- Additional work term and heat transfer
- Fractional steps strategy applied successfully
- Identified important processes needing future work (EOS in tension and fracture)